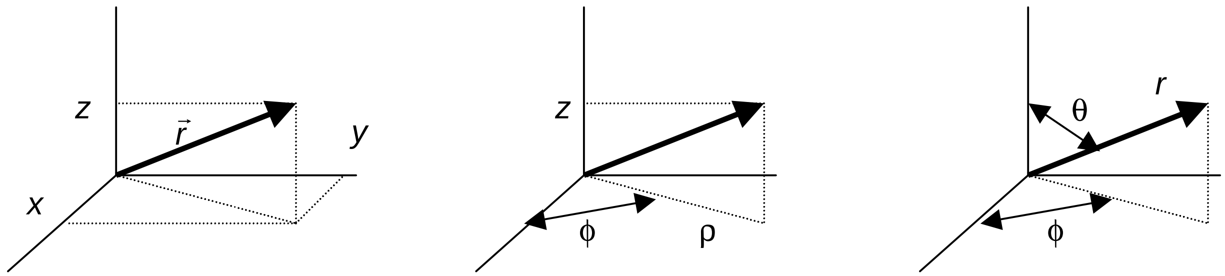
Cartesian, Cylindrical Polar, and Spherical Polar Coordinates



	Cartesian	Cylindrical Polar	Spherical Polar
$x =$ $y =$ $z =$	x y z	$\rho\cos\phi$ $\rho\sin\phi$ z	$r\sin\theta\cos\phi$ $r\sin\theta\sin\phi$ $r\cos\theta$
Unit Vectors (orthonormal)	$\hat{e}_x = \hat{i}$ $\hat{e}_y = \hat{j}$ $\hat{e}_z = \hat{k}$	$\hat{e}_\rho = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$ $\hat{e}_\phi = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$ $\hat{e}_z = \hat{k}$	$\hat{e}_r = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$ $\hat{e}_\theta = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$ $\hat{e}_\phi = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$
Infinitesimal Displacement	$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$	$d\vec{r} = d\rho\,\hat{e}_\rho + \rho\,d\phi\,\hat{e}_\phi + dz\,\hat{e}_z$	$d\vec{r} = dr\,\hat{e}_r + r\,d\theta\,\hat{e}_\theta + r\sin\theta\,d\phi\,\hat{e}_\phi$
$\lvert d\vec{r}\rvert^2 \equiv (ds)^2$	$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2$	$(ds)^2 = (d\rho)^2 + \rho^2(d\phi)^2 + (dz)^2$	$(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\phi)^2$
Volume Element	$dV = dx\,dy\,dz$	$dV = \rho\,d\rho\,d\phi\,dz$	$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

Vector Operators in Cartesian, Cylindrical Polar, and Spherical Polar Coordinates

In the table below, Φ is a scalar function of the spatial coordinates (not to be confused with the azimuthal angle ϕ) and $\vec{a}$ is a vector field, i.e. a vector whose components depend on the spatial coordinates. The vectors $\hat{i}, \hat{j}, \hat{e}_\rho, \hat{e}_\theta$ etc. are unit vectors pointing in the direction of increasing values of the respective coordinates.

Operator	Cartesian	Cylindrical Polar	Spherical Polar
$\vec{\nabla}\Phi$	$\frac{\partial\Phi}{\partial x}\hat{i}+\frac{\partial\Phi}{\partial y}\hat{j}+\frac{\partial\Phi}{\partial z}\hat{k}$	$\frac{\partial\Phi}{\partial\rho}\hat{e}_\rho+\frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}\hat{e}_\phi+\frac{\partial\Phi}{\partial z}\hat{e}_z$	$\frac{\partial\Phi}{\partial r}\hat{e}_r+\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{e}_\theta+\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\hat{e}_\phi$
$\vec{\nabla}\cdot\vec{a}$	$\frac{\partial a_x}{\partial x}+\frac{\partial a_y}{\partial y}+\frac{\partial a_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho a_\rho\right)+\frac{1}{\rho}\frac{\partial a_\phi}{\partial\phi}+\frac{\partial a_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 a_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, a_\theta\right)+\frac{1}{r\sin\theta}\frac{\partial a_\phi}{\partial\phi}$
$\vec{\nabla}\times\vec{a}$	$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ a_x & a_y & a_z\end{vmatrix}$	$\frac{1}{\rho}\begin{vmatrix}\hat{e}_\rho & \rho\hat{e}_\phi & \hat{e}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ a_\rho & \rho a_\phi & a_z\end{vmatrix}$	$\frac{1}{r^2\sin\theta}\begin{vmatrix}\hat{e}_r & r\hat{e}_\theta & r\sin\theta\,\hat{e}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ a_r & r a_\theta & r\sin\theta\, a_\phi\end{vmatrix}$
$\nabla^2\Phi$	$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$

