Hyperbolic Functions

Definitions of the hyperbolic functions:

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right) \qquad \sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\tanh x = \frac{1}{\coth x} = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\text{sech}\, x = \frac{1}{\cosh x} \qquad \text{cosech x} = \frac{1}{\sinh \text{x}}$$

Hyperbolic functions of imaginary arguments yield trig functions and *vice versa*:

$$\cosh(ix) = \frac{1}{2}\left(e^{ix} + e^{-ix}\right) = \cos x \qquad \sinh(ix) = i \sin x$$

$$\cos(ix) = \cosh x \qquad \sin(ix) = i \sinh x$$

Identity: $\cosh^2 x - \sinh^2 x = 1$

Inverse hyperbolic functions:

If $x = \sinh y$, then $y = \sinh^{-1} x$, etc.

$$\sinh^{-1} x = \ln\left(\sqrt{x^2+1} + x\right) \qquad \cosh^{-1} x = \ln\left(\sqrt{x^2-1} + x\right)$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

Calculus of hyperbolic functions:

$$\frac{d}{dx}(\cosh x) = \frac{1}{2}\left(e^{x} - e^{-x}\right) = \sinh x \qquad \frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}\left[\cosh^{-1}\left(\frac{x}{a}\right)\right] = \frac{d}{dx}\left[\sqrt{\left(\frac{x}{a}\right)^2 - 1} + \left(\frac{x}{a}\right)\right] = \frac{1}{\sqrt{x^2 - a^2}} \qquad \frac{d}{dx}\left[\sinh^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{\sqrt{x^2 + a^2}}$$