Eigenvectors and Eigenvalues

$(\mathbf{A} - \lambda\mathbf{I})\,\mathbf{x} = 0$ where **x** is an eigenvector and λ is the corresponding eigenvalue.

If eigenvalues λ_i and λ_j are distinct, ($\lambda_i \neq \lambda_j$), eigenvectors are orthogonal: $\langle x^i | x^j \rangle = 0$.

If k eigenvectors $\boldsymbol{x}^i$ have <u>same</u> eigenvalue (eigenvalue is ***k*-fold degenerate**), form linear combinations of the $\boldsymbol{x}^i$ that form an orthogonal set, e.g by Gram-Schmidt orthogonalization.

Mutually orthogonal eigenvectors form a complete basis for the $N \times N$ vector basis.

Eigenvalues of Hermetian matrices are **real**.

Eigenvalues of a unitary matrix have modulus 1: $|\lambda|^2 = 1$.

Operators (matrices) that commute (**AB** = **BA**) have eigenvectors in common.

Finding Eigenvalues and Eigenvectors

Eigenvalues are roots (λ_i) of secular determinant equation: $|\mathbf{A} - \lambda\mathbf{I}| = 0$.

$$\mathbf{Tr}\ A = \sum_{i=1}^{N} \lambda_i\ .$$

To find eigenvector $\mathbf{x}^i$ corresponding to eigenvalue λ_i: solve $(\mathbf{A} - \lambda_i\mathbf{I})\,\mathbf{x}^i = 0$ to get components of $\mathbf{x}^i$.

<u>Example (3 × 3)</u>:

$$\begin{aligned}
(A_{11} - \lambda_i)x_1^i + A_{12}x_2^i + A_{13}x_3^i &= 0 \\
A_{21}x_1^i + (A_{22} - \lambda_i)x_2^i + A_{23}x_3^i &= 0 \\
A_{31}x_1^i + A_{32}x_2^i + (A_{33} - \lambda_i)x_3^i &= 0
\end{aligned}$$