Special Square Matrices

Diagonal Matrices

$$A_{ij} = 0 \text{ unless } i = j: \quad \mathbf{A} = \begin{pmatrix} A_{11} & 0 & 0 & \cdot & \cdot & 0 \\ 0 & A_{22} & 0 & \cdot & \cdot & 0 \\ 0 & 0 & A_{33} & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & A_{NN} \end{pmatrix}$$

$$\det \mathbf{A} = |\mathbf{A}| = A_{11} \cdot A_{22} \cdots A_{NN}$$

$$\text{Inverse of diagonal matrix: } \mathbf{A}^{-1} = \begin{pmatrix} 1/A_{11} & 0 & 0 & \cdot & \cdot & 0 \\ 0 & 1/A_{22} & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1/A_{33} & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/A_{NN} \end{pmatrix}$$

If **A** and **B** are diagonal matrices, they commute, i.e. **AB** = **B**A.

Triangular Matrices

$$\begin{pmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ A_{N1} & A_{N2} & A_{N3} \cdot & \cdot & \cdot & A_{NN} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdot & \cdot & A_{1N} \\ 0 & A_{22} & A_{23} & \cdot & \cdot & A_{2N} \\ 0 & 0 & A_{22} & \cdot & \cdot & A_{3N} \\ 0 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & A_{NN} \end{pmatrix}$$

$$\det \mathbf{A} = |\mathbf{A}| = A_{11} \cdot A_{22} \cdots A_{NN}$$

Symmetric and Anti-Symmetric Matrices

Symmetric matrix: $\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdot & \cdot & A_{1N} \\ A_{12} & A_{22} & A_{23} & \cdot & \cdot & A_{2N} \\ A_{13} & A_{23} & A_{22} & \cdot & \cdot & A_{3N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{1N} & A_{2N} & A_{3N} & \cdot & \cdot & A_{NN} \end{pmatrix}$

Anti- Symmetric matrix: $\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdot & \cdot & A_{1N} \\ -A_{12} & A_{22} & A_{23} & \cdot & \cdot & A_{2N} \\ -A_{13} & -A_{23} & A_{22} & \cdot & \cdot & A_{3N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -A_{1N} & -A_{2N} & -A_{3N} & \cdot & \cdot & A_{NN} \end{pmatrix}$

Orthogonal Matrices

$\mathbf{A}^{T} = \mathbf{A}^{-1} \qquad \mathbf{A}^{T}\mathbf{A} = \mathbf{I} \qquad \det \mathbf{A} = \pm 1$

Note: If **A** is an orthogonal matrix and **y** = **Ax**, $\langle \mathbf{y} | \mathbf{y} \rangle = \langle \mathbf{x} | \mathbf{x} \rangle$**,** i.e. norm of a vector is invariant under rotation by an orthogonal matrix.

Hermetian, Anti-Hermetian, Unitary and Normal Matrices

Hermetian: $\mathbf{A}^{+} = \mathbf{A}$

Anti-Hermetian: $\mathbf{A}^{+} = -\mathbf{A}$

Unitary: $\mathbf{A}^{+} = \mathbf{A}^{-1}$

Normal: $\mathbf{A}\mathbf{A}^{+} = \mathbf{A}^{+}\mathbf{A}$ (Hermetian and unitary matrices are **normal**.)

