Resonance Transmittance Through a Metal Film With Subwavelength Holes

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***Abstract*—An analytical theory for extraordinary light transmittance through an optically thick metal film with subwavelength holes is developed. It is shown that the film transmittance has sharp peaks that are due to the Maxwell–Garnet resonances in the holes. There are localized electric and magnetic resonances resulting in, respectively, dramatically enhanced electric and magnetic fields in the holes. A simple analytical expression for the resonance transmittance is derived that holds for arbitrary hole distribution. It is also shown that there are other types of transmittance resonances, when the holes are arranged into a regular lattice. These resonances occur because of the excitation of surface plasmon polaritons propagating over the film surface. A combination of the two kinds of resonances results in a rich spectral behavior in the extraordinary optical transmittance.**

I. Introduction

IN THIS PAPER, we consider surface electromagnetic waves and the extraordinary light transmittance through an optically thick metal film which is perforated with subwavelength-size holes. In the optical and infrared spectral ranges, the excitation of the electron density coupled to the electromagnetic field results in a surface plasmon polariton (SPP) traveling on the metal surface (see, e.g., [1]–[3]). At the metal–air interface, the SPP is an H wave, with the direction of the magnetic field $\mathbf{H}$ parallel to the metal surface [3]. In the direction perpendicular to the interface, SPP's exponentially decay in both media. The SPP can propagate not only on the metal surface but also on the surface of artificial electromagnetic crystals, for example, on wire-mesh crystals [4]–[7]. This is because the real part of the *effective* dielectric constant can be negative in these mesa structures.

Since the SPP propagation includes rearrangement of the electron density, it is not surprising that its speed is less than the speed of light. As a result, the SPP cannot be excited by an electromagnetic wave impinging on a perfectly flat metal surface. The situation, however, changes when the film is modulated. In this case, the EM field inside the film is also modulated. When one of the spatial periods of the modulation coincides with the wavelength of the SPP, the latter can be excited by a normally incident EM wave.

In this paper, we show that the transmittance through a metal film with subwavelength holes (see Fig. 1) has sharp resonances corresponding to the excitation of various surface waves. Some of these waves are similar to the propagating SPP, whereas others represent the localized surface waves that are specific for a metal film with holes and were not discussed in the literature. The extraordinary optical transmittance (EOT) was first discovered in a seminal work [8] and then was intensively investigated (see, for example, [9]–[15]). A number of various models (with most of them being numerical simulations) were suggested to explain the EOT [14], [16]–[20]. Despite the sophisticated simulation codes used, the physical picture of the EOT is not fully understood. In this paper, we use a new analytical approach referred to as the generalized Ohms' law (GOL) [7], [18]. This approach allows us to develop a physical model, which provides a simple qualitative picture for the field distributions and the EOT.

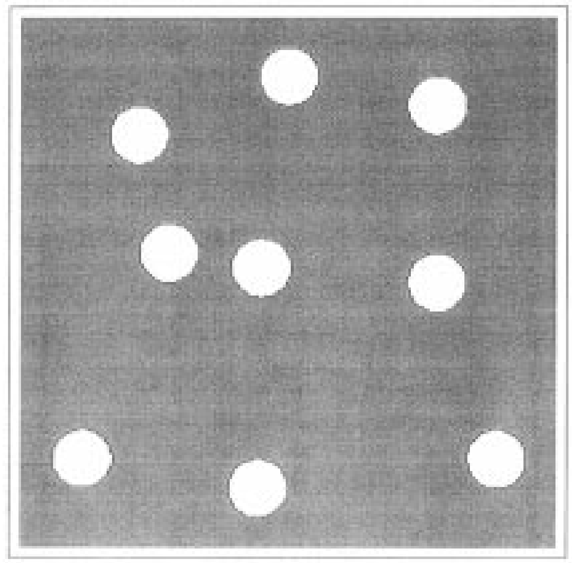

Fig. 1. Holes in a metal film.

The rest of the paper is organized as follows. First, we briefly describe the GOL approximation and extend it to the case of an optically thick metal film. Then we show our results for the local EM fields and the EOT. Finally, we discuss theoretical results obtained and compare them with experimental observations.

II. GOL Approximation

A new analytical approach to the calculation of optical properties of metal dielectric films, referred to as the GOL approximation, has recently been proposed [7], [18], [21]–[25]. First, in a review of the GOL approximation, we restrict ourselves to the case where all the external fields are parallel to the plane of the

Manuscript received August 31, 2001; revised March 11, 2002.

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Publisher Item Identifier S 0018-9197(02)05708-1.

film (normal incidence). It is supposed that a metal film, with possible holes, voids, or other inhomogeneities, is placed in the xy plane so that the z axis is perpendicular to the film, which has a thickness h. The external electromagnetic wave is incident onto the $z=-h/2$ interface of the film (front interface) and the transmitted wave is emitted from the $z=h/2$ interface (back interface). A typical spatial scale D of the film defects is supposed to be much smaller than the wavelength λ, i.e., $D \ll \lambda$; for cylindrical holes, D is the cylinder diameter. (In some graphs below we also show results for $D<\lambda$ that should be considered as an extrapolation.)

We consider first the electric and magnetic fields in close vicinity to the film. Namely, the electric and magnetic fields are considered at a distance a in front of the film $\mathbf{E}_1(\mathbf{r}) = \mathbf{E}(\mathbf{r}, -h/2-a)$, $\mathbf{H}_1(\mathbf{r}) = \mathbf{H}(\mathbf{r}, -h/2-a)$, and at the distance a behind the film $\mathbf{E}_2(\mathbf{r}) = \mathbf{E}(\mathbf{r}, h/2+a)$, $\mathbf{H}_2(\mathbf{r}) = \mathbf{H}(\mathbf{r}, h/2+a)$. All the fields and currents are monochromatic fields, with the usual $\exp(-i\omega t)$ time dependence. The vector $\mathbf{r}=\{x, y\}$ is a two-dimensional (2-D) vector in the xy plane. In the case of laterally inhomogeneous films, the average electric displacement current $\mathbf{D}(\mathbf{r}) = \int_{-h/2-a}^{h/2+a} \mathbf{D}(\mathbf{r}, z)\, dz = \int_{-h/2-a}^{h/2+a} \varepsilon(\mathbf{r}, z)\, \mathbf{E}(\mathbf{r}, z)\, dz$ and the average magnetic induction $\mathbf{B}(\mathbf{r}) = \int_{-h/2-a}^{h/2+a} \mathbf{B}(\mathbf{r}, z)\, dz = \int_{-h/2-a}^{h/2+a} \mu(\mathbf{r}, z)\mathbf{H}(\mathbf{r}, z)\, dz$ are functions of the vector $\mathbf{r}$. We assume hereafter, for simplicity, that permittivity ε and magnetic permeability μ are both scalars. In the GOL approximation, it is supposed that the local electromagnetic field is a superposition of two plane waves propagating in the $+z$ and $-z$ directions. This superposition of two waves is, indeed, different in different regions of the film. We neglect scattered and evanescent waves that propagate in the xy plane and have small amplitudes, $\sim(\lambda/D)^2$. Note that this estimate, however, does not hold for the resonant SPP discussed at the end of the paper. In the absence of the resonant SPP, we use the two-wave approximation when the electric $\mathbf{E}(\mathbf{r}, z)$ and magnetic $\mathbf{H}(\mathbf{r}, z)$ fields have their components in the $\{x, y\}$ plane only. Therefore, z components of $\operatorname{curl}\mathbf{E}(\mathbf{r}, z) \propto \mathbf{H}(\mathbf{r}, z)$ and $\operatorname{curl}\mathbf{H}(\mathbf{r}, z) \propto \mathbf{E}(\mathbf{r}, z)$ are zero. Then, the Maxwell equations $\operatorname{curl}\mathbf{E}(\mathbf{r}, z) = ik\mathbf{B}(\mathbf{r}, z)$ and $\operatorname{curl}\mathbf{H}(\mathbf{r}, z) = -ik\mathbf{D}(\mathbf{r}, z)$, when integrated from $z=-h/2-a$ to $z=h/2+a$, take the following form:

$$\begin{aligned}[\mathbf{n}\times(\mathbf{E}_2(\mathbf{r})-\mathbf{E}_1(\mathbf{r}))] &= ik\mathbf{B}(\mathbf{r}),\\ [\mathbf{n}\times(\mathbf{H}_2(\mathbf{r})-\mathbf{H}_1(\mathbf{r}))] &= -ik\mathbf{D}(\mathbf{r})\end{aligned} \tag{1}$$

where k is the wave vector, $\mathbf{n}=\{0, 0, 1\}$ is the unit vector normal to the plane of the film, and

$$\begin{aligned}\mathbf{E}_1(\mathbf{r}) &\equiv \mathbf{E}(\mathbf{r}, -h/2-a), \quad \mathbf{H}_1(\mathbf{r}) \equiv \mathbf{H}(\mathbf{r}, -h/2-a)\\ \mathbf{E}_2(\mathbf{r}) &\equiv \mathbf{E}(\mathbf{r}, h/2+a), \quad \mathbf{H}_2(\mathbf{r}) \equiv \mathbf{H}(\mathbf{r}, h/2+a)\end{aligned}$$

are 2-D vectors defined in the $\{x, y\}$ plane. The vectors $\mathbf{E}_1(\mathbf{r})$, $\mathbf{E}_2(\mathbf{r})$, $\mathbf{H}_1(\mathbf{r})$, and $\mathbf{H}_2(\mathbf{r})$ are curl-free since z components of $\operatorname{curl}\mathbf{E}(\mathbf{r}, z)$ and $\operatorname{curl}\mathbf{H}(\mathbf{r}, z)$ are zero. It is convenient to introduce the fields $\mathbf{E}=\mathbf{E}_1+\mathbf{E}_2$ and $\mathbf{H}=\mathbf{H}_1+\mathbf{H}_2$ that are also curl-free as follows:

$$\operatorname{curl}\mathbf{E}(\mathbf{r}) = 0, \quad \operatorname{curl}\mathbf{H}(\mathbf{r}) = 0. \tag{2}$$

The conservation laws give $\operatorname{div}\mathbf{D}(\mathbf{r}) = 0$ and $\operatorname{div}\mathbf{B}(\mathbf{r}) = 0$. For simplicity, we consider films having the mirror symmetry with respect to reflection in the $z=0$ plane. For such films, the displacement $\mathbf{D}(\mathbf{r})$ is a symmetric function of the fields $\mathbf{E}_1(\mathbf{r})$ and $\mathbf{E}_2(\mathbf{r})$. Therefore, we can write that

$$\mathbf{D}(\mathbf{r}) = u(\mathbf{r})\mathbf{E}(\mathbf{r})/k \tag{3}$$

where $u(\mathbf{r})$ is a dimensionless "Ohmic" parameter. A similar equation holds for the magnetic induction

$$\mathbf{B}(\mathbf{r}) = v(\mathbf{r})\mathbf{H}(\mathbf{r})/k \tag{4}$$

where $v(\mathbf{r})$ is another Ohmic parameter. The above equations have the form which is typical for constitutive equations in electrodynamics, but include parameters u and v that depend on the local geometry of the film.

To find the optical properties of the film, such as transmittance and reflectance, we average (1) over the film plane $\{x, y\}$ and introduce the effective film parameters, u_e and v_e, through the relations $u_e\langle\mathbf{E}\rangle = \langle u\mathbf{E}\rangle$ and $v_e\langle\mathbf{H}\rangle = \langle v\mathbf{H}\rangle$. Thus, we obtain the "integral" Maxwell equations for the film in the following form:

$$\begin{aligned}[\mathbf{n}\times(\langle\mathbf{E}_2\rangle-\langle\mathbf{E}_1\rangle)] &= iv_e\langle\mathbf{H}\rangle\\ [\mathbf{n}\times(\langle\mathbf{H}_2\rangle-\langle\mathbf{H}_1\rangle)] &= -iu_e\langle\mathbf{E}\rangle\end{aligned} \tag{5}$$

which relate the average fields from both sides of the film. We suppose that the wave enters the film from $z<0$, so that its amplitude is proportional to e^{ikz}. The incident wave is partially reflected and partially transmitted through the film. The electric field amplitude in the $z<0$ half-space, away from the film, can be written as $\tilde{E}_1(z) = e^{ikz} + re^{-ikz}$, where r is the reflection amplitude. Well behind the film, the electric component of the electromagnetic wave acquires the form $\tilde{E}_2(z) = te^{ikz}$, where t is the transmission amplitude. In the planes $z=-h/2-a$ and $z=h/2+a$, the average electric field equals $\langle E_1\rangle$ and $\langle E_2\rangle$, respectively. The electric field in the wave is matched with the average fields in the planes $z=-h/2-a$ and $z=h/2+a$, i.e., $\langle E_1\rangle = \tilde{E}_1(-h/2-a) = e^{-ik(h/2+a)} + re^{ik(h/2+a)}$ and $\langle E_2\rangle = \tilde{E}_2(h/2+a) = te^{ik(h/2+a)}$. The same matching for the magnetic fields gives $\langle H_1\rangle = e^{-ik(h/2+a)} - re^{ik(h/2+a)}$ and $\langle H_2\rangle = te^{ik(h/2+a)}$, in the planes $z=-h/2-a$ and $z=h/2+a$, respectively. The substitution of these expressions for the fields $\langle E_1\rangle$, $\langle E_2\rangle$, $\langle H_1\rangle$, and $\langle H_2\rangle$ in (5) gives two linear equations for t and r. By solving these equations, we obtain the reflectance and transmittance in the following form:

$$\begin{aligned}R \equiv |r|^2 &= \left|\frac{(u_e-v_e)}{(i+u_e)(i+v_e)}\right|^2\\ T \equiv |t|^2 &= \left|\frac{1+u_ev_e}{(i+u_e)(i+v_e)}\right|^2.\end{aligned} \tag{6}$$

Thus, the effective Ohmic parameters u_e and v_e completely determine the optical properties of inhomogeneous films. In the presence of holes, the EM field in front of and behind the film, in general, strongly depends on the distance from the film. Yet, if the spacing between the holes is smaller than the wavelength λ, the field recovers the form of a plane wave at large enough distances from the film. If we know the exact values of the electric and magnetic fields in some plane close to the film, then, the EM field in the whole half-space (for the corresponding side of the film) is determined unambiguously from Maxwell's equations. Thus, the zero-order diffraction is proportional to the EM field averaged in the midplane. In the GOL approximation, we consider variations of the local fields in two planes placed at some intermediate distance a from the film on both its sides. In these midplanes, we average the local fields and match them to the incident and reflected (in front of the film) and transmitted (behind the film) waves, respectively. The distance a to these planes is considered as a fitting parameter for the GOL approximation. The local fields fluctuate strongly as a function of the position in the mid-planes since they are similar to the fluctuations of the local EM field inside the metal film and in the holes. The average of the local fields, which represents the zero-order diffraction wave, includes all information about the field fluctuations that we need to calculate the transmitted and reflected waves. Therefore, using the average field over the midplanes at $z = -h/2 + a$ and $z = h/2 + a$ for calculating the reflectance and transmittance is not itself an approximation, even though the fields in these planes are calculated in the GOL approximation. Although the GOL is, indeed, an approximation, yet it yields analytical solutions and provides physical insight into a complicated problem. The GOL was shown to work well also for semicontinuous metal films where it describes the local-field distribution and reproduces well the experimentally observed anomalies in transmittance, reflection, absorption, and surface-enhanced Raman scattering (details can be found in [7]).

III. Transmittance of Nanoholes

Now we apply the developed GOL formalism to find the transmittance of a metal film with subwavelength holes. We find the effective parameters u_e and v_e for a film with holes from (3) and (4). Since electric $\mathbf{E}(\mathbf{r})$ and magnetic $\mathbf{H}(\mathbf{r})$ fields are curl-free in these equations [note also that $\operatorname{div}\mathbf{D}(\mathbf{r}) = 0$ and $\operatorname{div}\mathbf{B}(\mathbf{r}) = 0$], a number of efficient analytical and numerical methods, which were developed in the percolation theory can be used [7], [26]. Here we use the simplest approximation, namely, the Maxwell–Garnett (MG) approach that holds when the surface hole concentration is small, $p \ll 1$. In the MG approach the dipole approximation can be used that leads to the following expression for the electric field E_h in a hole

$$E_h = \frac{2E_m u_m}{u_m + u_h} \tag{7}$$

where u_m and u_h are the Ohmic parameters for the metal and holes, and the quantities $E_m = (E_1 + E_2)_m$ and $E_h = (E_1 + E_2)_h$ are the electric fields averaged over the metal and holes, respectively. From (7), we obtain the following expression for the "electric" effective parameter u_e:

$$u_e \equiv \frac{\langle uE\rangle}{\langle E\rangle} = \frac{(1-p)u_m E_m + p u_h E_h}{(1-p)E_m + pE_h}. \tag{8}$$

Repeating the same procedure, we find the "magnetic" effective parameter v_e, which is given by (8), with the following change $u_m \to v_m$ and $u_h \to v_h$. Note, that the retardation effects in the "z" direction are taken into account when we obtain equations for the Ohmic parameters. Therefore, they should not be accounted again in the MG equations. Note also that waves propagated in the "$x,\, y$" plane are neglected in the GOL approximation.

Now we substitute the parameters u_e and v_e in (6) and obtain the following expression for the transmittance

$$\begin{aligned} T &= \frac{16p^2|u_m^2(1+u_h v_h)|^2}{|\Sigma_1\Sigma_2|^2}\\ \Sigma_1 &= u_h - pu_h + (1+p)(1-iu_h)u_m - i(1-p)u_m^2\\ \Sigma_2 &= (i+u_m)(u_m v_h - 1) + p(i-u_m)(u_m v_h + 1) \end{aligned} \tag{9}$$

where we used the relation $u_m = -1/v_m$ that holds when the film thickness h is much larger than the metal skin depth δ, i.e., when $h \gg \delta$. Hereafter, we consider this case of a strong skin effect, which corresponds to most experiments with subwavelength holes reported so far.

The electric field in a hole raises formally up to infinity at $u_m \to -u_h$, if there are no losses. By substituting the $u_m = -u_h$ in (9), we obtain the following expression for the resonant transmittance $T = 4|u_m|/|1+u_m^2|$, which does not depend on the hole concentration p and, therefore, remains finite, even for $p \to 0$. When the magnetic resonance takes place, i.e., $v_m = -1/u_m = -v_h$, the resonant transmittance also remains finite at $p \to 0$. Thus, we conclude that the electric and magnetic MG resonances in the holes can result in the extraordinary optical transmittance.

To calculate the transmittance, we find the Ohmic parameters u_m, u_h, v_m, and v_h. We can obtain parameters u_m and v_m directly from solutions to the Maxwell's equations in the GOL approximation

$$u_m = -\cot(ak), \quad v_m = \tan(ak) \tag{10}$$

so that $u_m = -1/v_m$ (see [18]). To obtain the hole parameters u_h and v_h, we have to know the EM field distribution inside a hole. The inside field is a superposition of different eigenmodes for this subcritical waveguide. At the hole entrance, the internal field is similar to the plane wave, though its amplitude can be different significantly from the amplitude of an incident wave. (Note that we still neglect here the evanescent modes generated by the hole.) When we move deeper inside the hole, only the mode with the smallest eigenvalue survives. To simplify further qualitative considerations, we assume that the internal field is a plane wave near the entrance of the hole and it matches with the basic internal mode at the distance a from both ends of the hole. We use for this matching the same distance a as we used before to match local fields with the incident plane wave. As a result of such matching, we obtain

$$\begin{aligned} u_h &= \frac{k\tan(2ak) - \sqrt{\kappa^2-k^2}\tanh\left[\left(\frac{h}{2}-a\right)\sqrt{\kappa^2-k^2}\right]}{k + \sqrt{\kappa^2-k^2}\tan(2ak)\tanh\left[\left(\frac{h}{2}-a\right)\sqrt{\kappa^2-k^2}\right]}\\ v_h &= \frac{\sqrt{\kappa^2-k^2}\tan(2ak) + k\tanh\left[\left(\frac{h}{2}-a\right)\sqrt{\kappa^2-k^2}\right]}{\sqrt{\kappa^2-k^2} - k\tan(2ak)\tanh\left[\left(\frac{h}{2}-a\right)\sqrt{\kappa^2-k^2}\right]} \end{aligned} \tag{11}$$

where $\kappa = 3.68/D$ is the eigenvalue for the basic mode in a cylindrical waveguide, and D is the diameter of the hole. Note

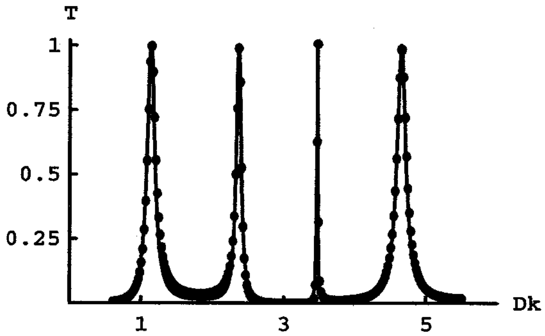

Fig. 2. Transmittance through “shallow” holes $(h < 2a)$; $a/D = 0.6$, $h/D = 0.8$, $p = 0.1$. The solid line is the resonance approximation [(12)]. The points represent calculations with (9).

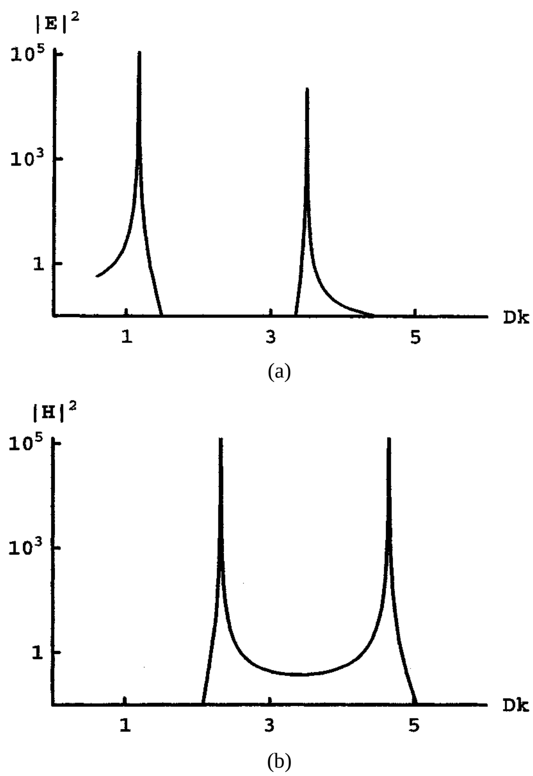

Fig. 3. (a) Electric and (b) magnetic field in a hole for the system with the same parameters as in Fig. 2. The incident field amplitude is set to be equal to one.

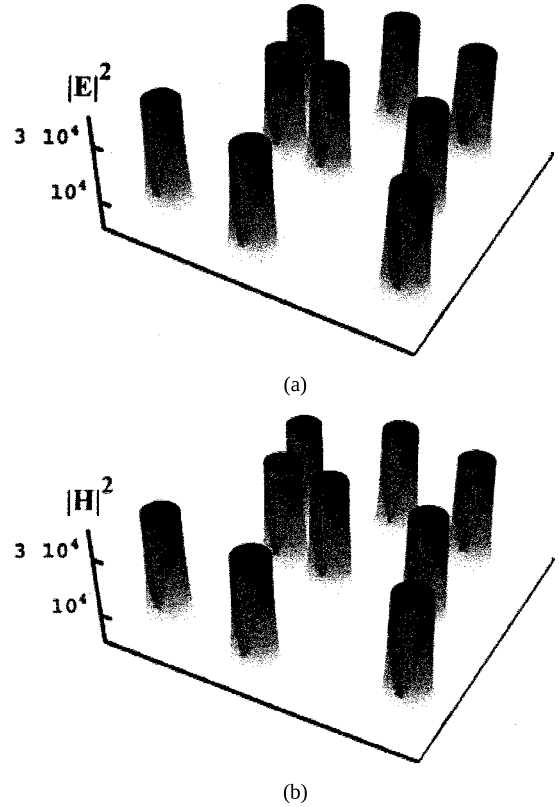

Fig. 4. Spatial distribution of (a) electric and (b) magnetic fields near the MG resonance for the system with the same parameters as in Fig. 2. (a) $kD = 0.992$. (b) $kD = 1.96$.

that the hole represents a subcritical waveguide, when $\lambda > 1.71D$ [3].

If holes are “shallow” enough so that $h < 2a$, the wave remains almost planar inside the hole and the Ohmic parameters can be simplified to $u_h = v_h = \tan[(a + (h/2))k]$. By substituting these expressions in (9) and considering the limit $p \ll 1$, we obtain the simple expression for the transmittance, shown in (12), at the bottom of the page. The transmittance thus obtained is shown in Fig. 2. The maximum position for T is a periodical function of k; the peak width depends on k. Some maxima can disappear when the corresponding numerators in (12) vanish. The odd resonances in (12) correspond to the maxima of the electric field in the holes, whereas the even resonances are due to the maxima in the magnetic field in the holes, as shown in Fig. 3. The spatial distribution of the fields near the resonance is shown in Fig. 4. For the considered lossless system, the electric and magnetic fields tend to infinity in the resonance. In any real metal film, the resonant fields acquire some finite values limited by losses. Yet, if the losses are relatively small because, for example, of a strong skin effect, the resonance field remains large, leading to the EOT, even for very small holes, with $D \ll \lambda$.

When losses are relatively small, they can be analytically taken into account for the case of shallow holes. The primary cause for losses is a finite refractive index for metal, which results in losses in the metal and losses inside the holes themselves. When the metal refractive index $n = \sqrt{-\varepsilon}$ is not infi-

$$T_r = \sum_j \frac{4p^2 \sin^4\left(\frac{2aj\pi}{4a+h}\right)}{4p^2 \sin^4\left(\frac{2aj\pi}{4a+h}\right) + (4a+h)^2\left(k - \frac{j\pi}{4a+h} + \frac{p}{4a+h}\sin\left(\frac{4aj\pi}{4a+h}\right)\right)^2} \tag{12}$$

nite, but still large, the metal Ohmic parameters are changed and have the following form:

$$u_m = -\cot(ak) + \frac{1}{n\cos^2(ak)}, \quad v_m = \tan(ak) + \frac{1}{n\sin^2(ak)}.$$

Losses in the holes in the limit of large n lead also to the renormalization of the wavevector: $k \to k + (1/nD)$, where D is the hole diameter. Taking into account the changes described above in the ohmic parameters of metal and the k renormalization, we obtain the expression for the transmittance, shown in (13), at the bottom of the page, where $n = n_1 - in_2 \equiv \sqrt{-\varepsilon}$ is metal refractive index. Note that the real part of the refractive index leads to the shift in the resonance positions, while the imaginary part leads to a decrease of the resonance magnitude.

For "deep" holes with $h > 2a$, the EM field decays exponentially inside a hole, as it should do for a long subcritical waveguide. In actual calculations we set the distance a from the reference plane, which is a single fitting parameter in the theory, as $a = 0.6D$, in agreement with our previous estimates for thin films [18], [21]–[25]. Therefore, the condition $h > 2a$ relates, in fact, the film thickness and the hole diameter. In the case of deep holes, as follows (9)–(11), the resonances lose their k periodicity. The positions of the electric and magnetic resonances, k_{jE} and k_{jH}, respectively, can be found, in this case, from the following equations:

$$k_{jE}\cot(3ak_{jE}) + \sqrt{\kappa^2 - k_{jE}^2}\tanh\left[\left(\frac{h}{2} - a\right)\sqrt{\kappa^2 - k_{jE}^2}\right] = 0 \quad (14)$$

$$\sqrt{\kappa^2 - k_{jH}^2}\,\tan(3ak_{jH}) + k_{jH}\tanh\left[\left(\frac{h}{2} - a\right)\sqrt{\kappa^2 - k_{jH}^2}\right] = 0. \quad (15)$$

As follows from these equations, the electric and magnetic resonances merge together with an increase of the film thickness h. Therefore, we have an interesting system here, with the electric and magnetic fields acquiring large values at almost the same points.

The transmittance for a metal film with deep holes can be represented in the following form

$$T = \sum_j \frac{4p^2\sin^4(2ak_j)}{(k - k_j + \Delta_j)^2\Gamma_j^2 + 4p^2\sin^4(2ak_j)} \quad (16)$$

where Δ_j and Γ_j are shown in (17) and (18) at the bottom of the next page. The resonant wavevector k_j in these equations takes values k_{jE} and k_{jH} given by (14) and (15), respectively. $T(k)$

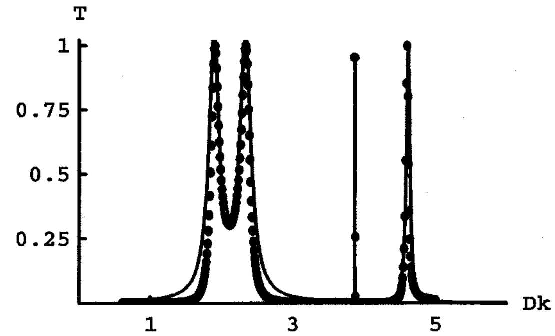

Fig. 5. Transmittance through "deep" holes $(h > 2a)$; $a/D = 0.6$, $h/D = 1.45$, $p = 0.1$. The solid line is the resonance approximation [(16)]; the points are results of calculations with (9).

thus obtained is shown in Fig. 5. We can see that the k behavior of the transmittance can be rather peculiar when the thickness of the film increases: the peaks, for example, can move and merge together. Note that (16) holds when the neighboring electric and magnetic maxima do not overlap much; otherwise, the general equation (9) should be used to calculate the transmittance, using the known Ohmic parameters u_m, u_h, v_m, and v_h.

The results obtained can be easily verified in the microwave range, because in this case it is far easier (than in the optical range) to control the system parameters and losses are less important. The proper holes can be drilled through, for example, a silver or aluminum slab (see, e.g., [27]). In the optical range, losses cannot be neglected, even for silver films providing the largest field enhancement. Losses become most important in the resonance when the local fields are strongly enhanced. To take into account losses in a deep hole we consider the hole as a waveguide with finite losses [3], by taking into account the actual silver dielectric permittivity (see [1], [28]–[30], , and references therein). When the skin depth δ is much smaller than the hole diameter D (for silver $\delta \sim 10$ nm in the optical range), the losses result in the appearance of the imaginary part in the wavevector k in (14) and (15).

Transmittance thus obtained for an array of nanoholes in a silver film is in qualitative agreement with the well-known experiments [9] and [12] as illustrated in Fig. 6. Our theory does predict that the long wavelength peaks in the extraordinary transmittance are not sensitive to the periodicity. This conclusion might look to be contradicting experimental observations [8] where the peak positions in the transmittance showed some dependence on the periodicity. Specifically, in [8], authors showed that, for two different samples, a silver film with $b_1 = 0.6$ μm, $D_1 = 0.15$ μm, $h_1 = 0.2$ μm (system 1) and a gold film with $b_2 = 1.0$ μm, $D_2 = 0.350$ μm, $h_2 = 0.3$ μm (system

$$T_r(k) = \sum_j \frac{4p^2\sin^4\left(\frac{2aj\pi}{4a+h}\right)}{4\left[p\sin^2\left(\frac{2aj\pi}{4a+h}\right) - \frac{n_2}{n_1^2}\right]^2 + \frac{(4a+h)^2 n_2^2}{D^2 n_1^4} + (4a+h)^2(k-k_j)^2}$$

$$k_j = \frac{j\pi}{4a+h} - \frac{2}{(4a+h)n_1} - \frac{1}{Dn_1} - \frac{p}{4a+h}\sin\left(\frac{4aj\pi}{4a+h}\right) \quad (13)$$

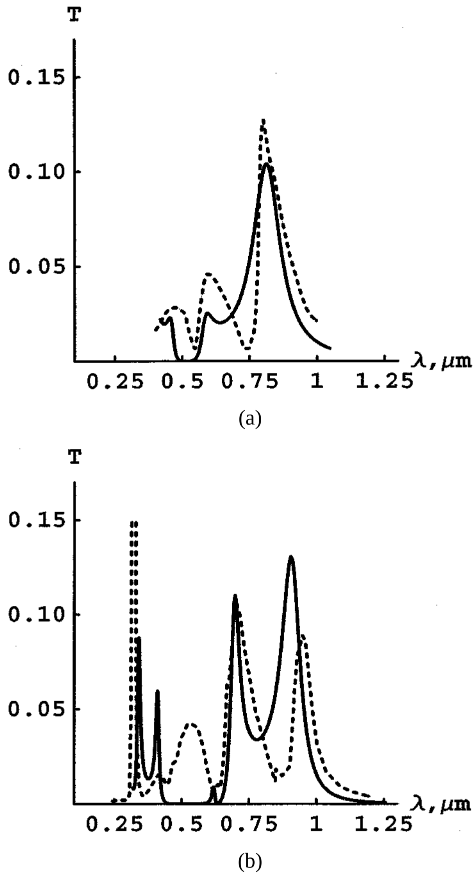

Fig. 6. Extraordinary optical transmittance through a regular array of holes in two different silver films: (a) a free-standing silver film and (b) a silver film on a quartz substrate. The dashed lines represent experimental data of (a) [12] and (b) [9]. The solid lines show results of the theory. The parameters used are as follows: (a) $a = 0.17$ μm, $D = 0.28$ μm, $h = 0.32$ μm, $b = 0.75$ μm and (b) $a = 0.9$ μm, $D = 0.15$ μm, $h = 0.2$ μm, $b = 0.6$ μm, $p = 0.049$.

2), the peak positions in the transmittance spectra almost coincide, when the transmittances are plotted as functions of λ/b, where b is the period of the hole array. One might think that this indicates a critical role of the periodicity. However, it is important to note that other parameters, namely, D and h (and materials used for the two samples—silver and gold) were also different for the two systems, so that the similar peak positions observed, as functions of λ/b, alone are not sufficient to prove the periodicity role. If we apply our theory (which does not rely on the periodicity) for the two sets of parameters used in the experiments, the ratio of the scaling factors $4a_i + h_i$ [see (12)] in the $T(k)$ dependence is given by $(4a_1 + h_1)/(4a_2 + h_2) \cong 0.5$, which is rather close to the ratio of $b_1/b_2 = 0.6$. Thus, our theory also predicts very similar positions for the transmittance maxima in the systems 1 and 2, and thus the theory does not contradict the experimental observations.

We also note that the peak positions in Fig. 6(a) and (b), if they were re-plotted as functions of b/λ (not shown), are well correlated, for the two silver films with different periods b. This occurs for both experimental and theoretical data despite the fact that the theory does not invoke any periodicity. Again, the similar peak positions (as functions of b/λ) for the two different films do not prove the role of periodicity because other important characteristics of the film, namely, the hole diameters and film thicknesses, are also different for these two films.

In the end, we mention that experiments of the same group of authors [11] show that the EOT can occur even for a set of only seven holes. Finally, in recent near-field experiments [15], strong enhancement of the local field has been observed for a single hole and a pair of holes; thus, the local field enhancement, which is needed for the EOT, does not require, in general, the periodicity.

IV. Discussion: Localized Versus Propagating Surface Plasmons

The results obtained above do not actually depend on the arrangement of the holes and require only that the surface concentration is small, $p \ll 1$. In particular, (12) and (16) hold for the holes arranged into a regular lattice since the MG approach works in this case as well (see [7]). Below we consider the square lattice, with period b so that the hole concentration $p = \pi D^2/4b^2$. There are new properties for the transmittance through the square array of holes in a metal film. Apart from the resonances given by (12) and (16), new resonances can appear, resulting from the excitation of propagating surface plasmon polaritons. The SPP is characterized by the wave vector k_p, in contrast to the localized resonant fields discussed above. When one of the spatial constants $B_{n_1 n_2} = b/\sqrt{n_1^2 + n_2^2}$ $(n_1, n_2 = 0, 1, 2 \ldots; n_1 \cdot n_2 \neq 0)$ of the lattice coincides with the SPP wavelength $\lambda_p = 2\pi/k_p$, the SPP is efficiently excited on the surface of the film. Since the film is optically thick the SPP is excited first on the front interface of the film. Yet, eventually, it spreads over *both* sides of the film, so that SPPs on both interfaces, front and back, of the film are excited. There is a straightforward analogy between the SPP on the two sides of the film and two identical oscillators coupled together. The coupling can be arbitrary weak, nevertheless, if we push the first oscillator, then, in some period of time (which depends on the coupling strength) the second oscillator starts to oscillate with the same amplitude as the first oscillator. By the same token, the two SPPs on the different sides of the film will eventually have the same amplitude in the absence of damping. When a

$$\Delta_j = \frac{k_j p (k_j^2 - \kappa^2) \sin(4ak_j)}{2(h+4a)k_j^3 - (h+10a)k_j\kappa^2 + \kappa^2[(h-2a)k_j \cos(6ak_j) + 2\sin(6ak_j)]} \tag{17}$$

$$\Gamma_j = \frac{2(h+4a)k_j^3 - (h+10a)k_j\kappa^2 + \kappa^2[(h-2a)k_j \cos(6ak_j) + 2\sin(6ak_j)]}{2\sqrt{2}k_j(k_j^2 - \kappa^2)} \tag{18}$$

SPP propagates on the back side of the film, it interacts with the holes and, as a result, converts its energy back to the plane wave reemitted from the film. Therefore, at the plasmon resonance, the film becomes almost transparent.

To take into account the SPP excitation, we repeat the procedure above to obtain the GOL equations for the SPP fields, using the radiative boundary conditions in the reference planes. Then, instead of (5) we obtain an infinite set of equations where the EM fields for SPPs and the incident wave are coupled through the fields inside the holes. According to the speculations above, only the interaction of the incident wave with the resonant SPP is important. Therefore, for a description of any particular SPP-induced resonance in the transmittance, the set of equations reduces to two pairs of equations: one for the incident wave and another for the resonant SPP.

As follows from the considerations above, the SPP interacts "twice" with the holes to result in the SPP-induced resonant transmittance; the "first interaction" with holes is needed to provide the coupling between the SPPs on the two sides of the film and the "second interaction" occurs when the back-side SPP is converted into the transmitted wave. Therefore, the transmittance amplitude t is proportional to p^2 and, as a result, the transmittance $T = |t|^2$ is proportional to p^4, as opposed to the p^2 dependence given by (12) and (16). Indeed, in Fig. 6(b), the peak near $\lambda \simeq 0.6$ μm, which we believe is associated with the SPP excitation, is rather small (the spatial period of the hole lattice was given as $b = 0.6$ μm in the experiment [9]). Note that the experimentally observed peak at $\lambda \simeq 0.6$ μm is larger in amplitude and much broader than the theoretical one. We believe that the discrepancy is due to imperfections on the surface of the film that are clearly seen in [9, Fig. 1]. These imperfections can increase the coupling of the incident wave to the SPPs (thus leading to the larger peak amplitude) and, simultaneously, result in the broadening of the resonance because of the ohmic and scattering losses. This problem is out of the scope of the present paper; a detailed theory, using the GOL approach for the surface plasmon polaritons on a film with periodic arrays of nanoholes will be presented elsewhere.

Above, we considered the case when a metal film is irradiated homogeneously by a plane electromagnetic wave. It is interesting to mention another possibility when only one of the holes is illuminated by a light source. This can be accomplished, for example, using a nanometer-size probe of a near-field scanning optical microscope. At resonance, electric (or/and magnetic) fields spread out from the illuminated hole toward other holes because of interactions between the holes via plasmons. Such holes can be arranged into any desired structure that can localize light and guide the propagation of the electromagnetic energy along the structures. Such nanoengineered structures can be used as integrated elements in various optoelectronic and photonic devices, including quite sophisticated ones, such as optical computers.

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