Ph.265. Equation solving, differentiation and integration with Maple.

Equations

We start with a solution of a single equation. The general equation can be written in the form

$$L(x)=R(x), \quad (1)$$

where L and R are some function of the variable x. To find the solution of the equation means to find such value of x that the functions L and R would produce same values. Using this definition of equation and solution we arrive to the following method of solving the arbitrary equation graphically. Namely, we use Maple to plot the graphs of the functions L and R for some region of x. If the graphs intersect, the abscissa of intersection corresponds to the solution of the Eq. (1).

It is also obvious that the solution of Eq. (1) is the same as the solution of:

$$L(x)-R(x)=0 \quad (2)$$

Note, that while the Eq.(2) involves only one function (L-R), its roots (values of x where L-R is equal to 0), coincide with the solutions of seemingly more general Eq.(1).

Maple can analytically solve some of the equations. To get a solution, one should enter `solve(equation, var)`.

We can also solve a set of equations. In this case the individual equations are combined into a set via curvy braces {}. Similarly, independent variables are also combined into the second set.

Another way to solve an equation is to convert it to a form of Eq.2, and then call the function `roots(fn)`. We receive a series of roots of fn along, with multiplicity of each root.

In some cases, neither Maple nor human can find the analytical solution of the equation. But the solution may exist! This solution can be either found graphically, or numerically. Numerical solutions can be found using `fsolve(fun, x=xMin..xMax)`.

Derivatives

Sometimes we need to find the derivative of the function. Maple function diff(f,x) does exactly that. However, we should remember that the argument x must be a variable! This means that if we want to calculate the derivative of sin at x=2, we cannot write `x:=2; diff(sin(x),x);` The workaround is using the function `D(f)`. Note that we do not specify the differentiation variable! It is possible to use function D to find partial and multiple (second, third, etc) derivatives. This is achieved by calling `D(f,i1,i2,i3)`, where `i1`, is the variable for first differentiation, `i2` – for the second one, etc. For example, to calculate the second derivative of sin, we would call `D(sin,1,1)`.

Sometimes, the function we want to explore cannot be represented in symbolic form (example – price of a given stock as a function of time). However, we can calculate the derivative of any function we can calculate. This is done using the definition of a derivative: we first select small number dx, and then calculate the derivative:

`(F(x+dx)-F(x))/dx`; in this case "small" means that the value of function `F` at `x` and `(x+dx)` should be approximately the same.

Integrals

Similarly to differentiation, we may use Maple to perform integration. The analytical integration is performed using `int(f(x),x)`; We can also calculate the definite integrals using `int(f(x),x=xIni..xFin);`

Note that the integration function `int` is similar to `diff` – meaning that in order to call this function x must be a variable with no assigned value. If we definitely want to assign the value to x after integration we must call `unapply(int(f(x),x),x);`