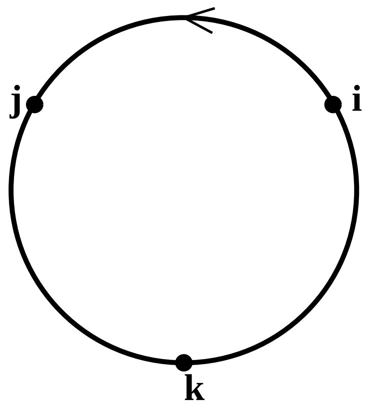


Figure 3.4: The cross product multiplication table.

Using an orthonormal basis such as $\{\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}\}$, the geometric formula reduces to the standard component form of the cross product. If $\vec{\boldsymbol{v}} = v_x\,\hat{\boldsymbol{\imath}} + v_y\,\hat{\boldsymbol{\jmath}} + v_z\,\hat{\boldsymbol{k}}$ and and $\vec{\boldsymbol{w}} = w_x\,\hat{\boldsymbol{\imath}} + w_y\,\hat{\boldsymbol{\jmath}} + w_z\,\hat{\boldsymbol{k}}$, then [5]

$$\begin{aligned} \vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}} &= (v_x\,\hat{\boldsymbol{\imath}} + v_y\,\hat{\boldsymbol{\jmath}} + v_z\,\hat{\boldsymbol{k}}) \times (w_x\,\hat{\boldsymbol{\imath}} + w_y\,\hat{\boldsymbol{\jmath}} + w_z\,\hat{\boldsymbol{k}}) \\ &= (v_y w_z - v_z w_y)\,\hat{\boldsymbol{\imath}} + (v_z w_x - v_x w_z)\,\hat{\boldsymbol{\jmath}} + (v_x w_y - v_y w_x)\,\hat{\boldsymbol{k}} \end{aligned} \tag{3.14}$$

which is often written as the symbolic determinant

$$\vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix} \tag{3.15}$$

We emphasize that this works in *any* (right-handed) orthonormal basis. In cylindrical coordinates, not only is

$$\hat{\boldsymbol{r}} \times \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{z}} \tag{3.16}$$

but cross products can be computed as

$$\vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}} = \begin{vmatrix} \hat{\boldsymbol{r}} & \hat{\boldsymbol{\phi}} & \hat{\boldsymbol{z}} \\ v_r & v_\phi & v_z \\ w_r & w_\phi & w_z \end{vmatrix} \tag{3.17}$$

[5] This argument uses the distributive property, which must be proved geometrically if one starts with (3.9) and the right-hand rule. This is straightforward in 2 dimensions, but somewhat more difficult in 3 dimensions [10].