

Capstones in Physics: Electromagnetism

3. Magnetostatics

3.1. Lorentz Force Law

3.2. Biot-Savart law and Ampere's law

3.3. Magnetic vector potential

3.4. Magnetic dipole

3.5. Magnetization

3.1 LORENTZ FORCE LAW

Lorentz force on a charge Q is $\mathbf{F}(\mathbf{r}) = Q [\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})]$

Work done by the fields

when particle moves by $d\mathbf{r} = \mathbf{v} dt$ is

$$dW = d\mathbf{r} \cdot \mathbf{F}(\mathbf{r}) = Q dt [\mathbf{E}(\mathbf{r}) \cdot \mathbf{v} + \mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}(\mathbf{r}))] = Q dt \mathbf{E}(\mathbf{r}) \cdot \mathbf{v}$$

(vanishes, since $\mathbf{v} \perp \mathbf{v} \times \mathbf{B}$) †

IDEALIZED CURRENTS

THE IDEALIZATION OF THE WIRE

λ = line charge density

I = current

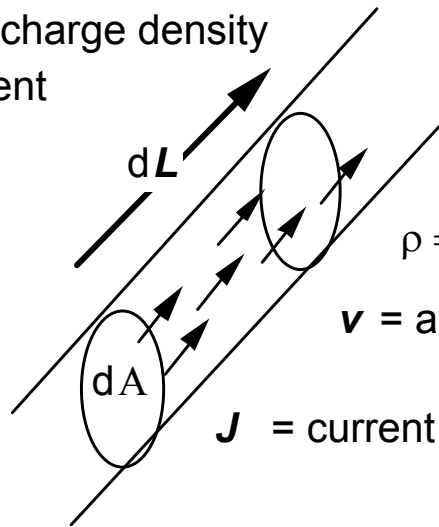
$$\mathbf{J} dA dL = I dL$$

$$\rho dA dL = \lambda dL$$

ρ = charge density

\mathbf{v} = average velocity

$$\mathbf{J} = \text{current density} = \rho \mathbf{v}$$



$$d\mathbf{F}_{\text{magnetic}} = I d\mathbf{L} \times \mathbf{B}(\mathbf{R}), \quad \mathbf{F}_{\text{magnetic}} = I \int d\mathbf{L} \times \mathbf{B}(\mathbf{R})$$

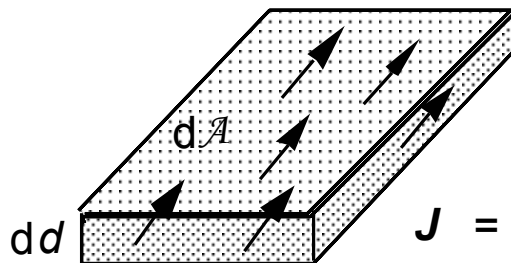
THE IDEALIZATION OF THE SHEET

\mathbf{K} = surface current density

σ = surface charge density

$$\mathbf{J} d\mathcal{A} dd = \mathbf{K} dA$$

$$\rho d\mathcal{A} dd = \sigma dA$$



ρ = charge density

\mathbf{v} = average velocity

$$\mathbf{J} = \text{current density} = \rho \mathbf{v}$$

$$d\mathbf{F}_{\text{magnetic}} = dA \mathbf{K} \times \mathbf{B}(\mathbf{R}), \quad \mathbf{F}_{\text{magnetic}} = \int dA \mathbf{K} \times \mathbf{B}(\mathbf{R})$$

3.2. BIOT-SAVART LAW AND AMPERE'S LAW

3.2.A. Biot-Savart law for \mathbf{B} due to current distribution \mathbf{J}

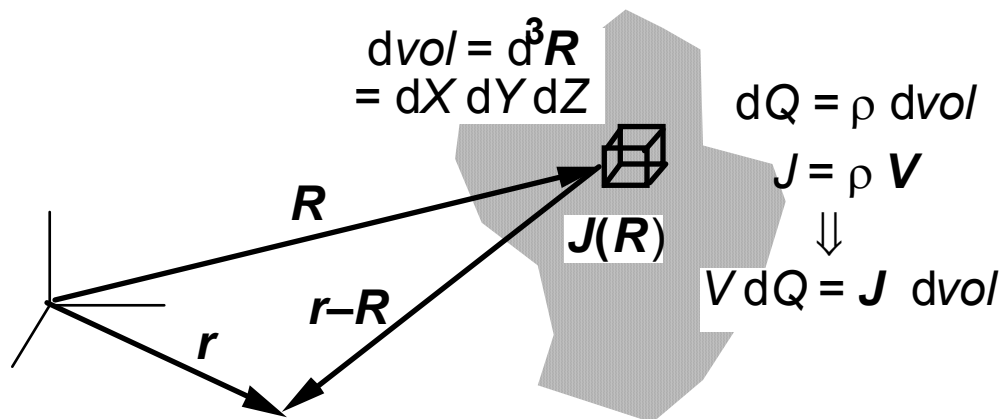
steady state case: assume charge density ρ and current \mathbf{J}
independent of time;

then \mathbf{E} and \mathbf{B} are also time independent

May also be useful approximation when slowly varying!

In this case the Maxwell equations for \mathbf{E} and \mathbf{B} decouple:

\mathbf{E} depends only on ρ , \mathbf{B} depends only on \mathbf{J}



for time-independent sources, $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3\mathbf{R} \mathbf{J}(\mathbf{R}) \times (\mathbf{r}-\mathbf{R}) / |\mathbf{r}-\mathbf{R}|^3$

proof: check $\nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

important special case: thin wire, cross section dA ,

total current $I = \mathbf{J} dA$

$d^3\mathbf{R} \mathbf{J}(\mathbf{R}) = dL dA \mathbf{J} = I d\mathbf{L}$ where $d\mathbf{L}$ = path element

BIOT-SAVART EXAMPLE

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int d\mathbf{L} \times (\mathbf{r} - \mathbf{R}) / |\mathbf{r} - \mathbf{R}|^3,$$

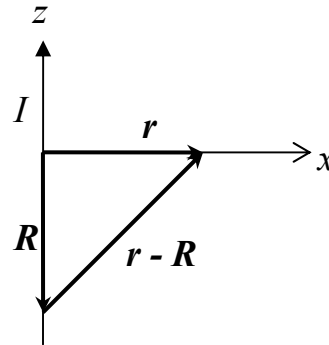
$$d\mathbf{L} = d\mathbf{Z} = \hat{\mathbf{z}} dZ,$$

$$\mathbf{R} = \mathbf{Z} = Z \hat{\mathbf{z}}$$

$$\mathbf{r} - \mathbf{R} = \mathbf{r} - Z \hat{\mathbf{z}},$$

$$\hat{\mathbf{z}} \times (\mathbf{r} - Z \hat{\mathbf{z}}) = \hat{\mathbf{z}} \times \mathbf{r},$$

$$|\mathbf{r} - \mathbf{R}|^2 = r_{\perp}^2 + Z^2$$



$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} (\hat{\mathbf{z}} \times \mathbf{r}) \int_{-\infty}^{\infty} dZ \frac{1}{(r_{\perp}^2 + Z^2)^{3/2}} = \frac{\mu_0 I}{4\pi} (\hat{\mathbf{z}} \times \mathbf{r}) \left(\frac{2}{r_{\perp}^2} \right) = \frac{\mu_0 I}{2\pi r_{\perp}} \hat{\phi}$$

check: $\oint d\mathbf{L} \cdot \mathbf{B} = \mu_0 I$ on a circular path centered on the wire

3.2.B. AMPERE'S LAW

The equation for the curl of \mathbf{B} : $\nabla \times \vec{B} = \mu_0 \vec{J}$

Stokes theorem : $\int (\nabla \times \vec{B}) \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a} = \mu_0 I$

A much simpler way to find \mathbf{B}

When the source distribution has lots of symmetry,

we can often use Ampere's law directly to find the magnetic field

Example: straight-line currents

we can argue from symmetry that the field $\mathbf{B}(\mathbf{r})$

- is independent of ϕ or z , so it depends only on $r_{\perp} + \rho$, $\mathbf{B}(\mathbf{r}) = \mathbf{B}(\rho)$
- is in the azimuthal direction, along $\hat{\phi}$ $\mathbf{B}(\rho) = \hat{\phi} B(\rho)$

To find the remaining unknown function $B(\rho)$,

we apply Ampere's law to a cleverly chosen path chosen so that it is

- either along the field direction, but with constant field, or
- perpendicular to the field

A circular path of radius ρ centered on the wire has $d\mathbf{L} = \hat{\phi} \rho d\phi$,

$$\oint d\mathbf{L} \cdot \mathbf{B} = \int_0^{2\pi} d\phi \rho \hat{\phi} \cdot \hat{\phi} B(\rho) = 2\pi\rho B(\rho) = \mu_0 I \Rightarrow B(\rho) = \frac{\mu_0 I}{2\pi\rho}$$

Example

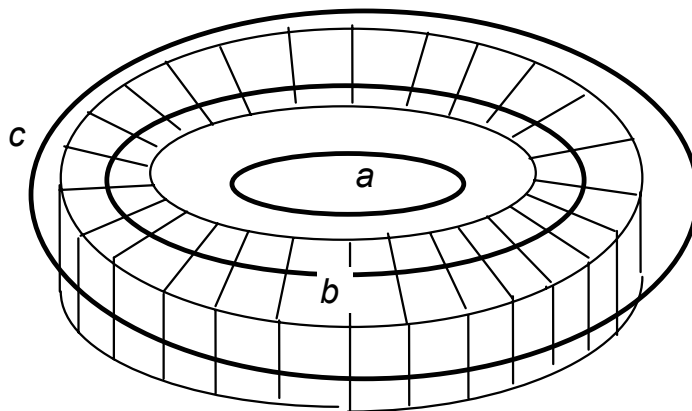
Toroidal coil

square cross section

N turns, current I

following Lorraine,
Corson & Lorraine

Apply Ampere's
circular law in integral
form to paths:



$$\oint d\mathbf{L} \cdot \mathbf{B} = \mu_0 \int dA \cdot \mathbf{J} = \mu_0 I_{\text{enclosed}}$$

paths a, b and c: $d\mathbf{L}$ is azimuthal, so $\oint d\mathbf{L} \cdot \mathbf{B} = 2\pi\rho B_\phi$

where ρ is the radius of the path

$$\oint d\mathbf{L} \cdot \mathbf{B} = \mu_0 \int dA \cdot \mathbf{J} = \mu_0 I_{\text{enclosed}}, \text{ so } B_\phi = \frac{\mu_0}{2\pi\rho} I_{\text{enclosed}}$$

path a:

can find a surface bounded by this path with no current penetrating it, so

$$B_\phi = 0.$$

path c: likewise

$$B_\phi = 0;$$

note the surface can be chosen at will, only its boundary is fixed.

For example, choose a plane surface,

then equal currents flow up and down through the surface

path b: current penetrating surface is NI , so

$$B_\phi = \frac{\mu_0}{2\pi\rho} NI$$

3.3. MAGNETIC VECTOR POTENTIAL

Because $\nabla \cdot \mathbf{B} = 0$,

there exists a vector function \mathbf{A} such that $\mathbf{B} = \nabla \times \mathbf{A}$

The ampere's law

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

You can add to the magnetic potential any function whose curl vanishes

with no effect on \mathbf{B} . $\Rightarrow \nabla \cdot \mathbf{A} = 0$

The Ampere's law becomes $\nabla^2 \vec{A} = -\mu_0 \vec{J}$: Poisson's equation.

Therefore,
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{R})}{|\vec{r} - \vec{R}|} d^3 \vec{R}$$

For line and surface currents

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I}}{|\vec{r} - \vec{R}|} dl = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}}{|\vec{r} - \vec{R}|}; \quad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{R})}{|\vec{r} - \vec{R}|} da$$

Multipole expansion of the vector potential

$$\begin{aligned} \vec{A}(\vec{r}) &= \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l}}{|\vec{r} - \vec{R}|} = \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint R^n P_n(\cos \alpha) d\vec{l} \\ &= \frac{\mu_0 I}{4\pi} \left[\frac{1}{r} \oint d\vec{l} + \frac{1}{r^2} \oint R \cos \alpha d\vec{l} + \dots \right] \end{aligned}$$

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0 I}{4\pi r^2} \oint R \cos \alpha d\vec{l} = \frac{\mu_0 I}{4\pi r^2} \oint \hat{r} \cdot \vec{R} d\vec{l} = -\frac{\mu_0 I}{4\pi r^2} \hat{r} \times \oint d\vec{a}$$

$$\Rightarrow \vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \quad \text{where } \vec{m} = I \oint d\vec{a} = I \vec{a}$$

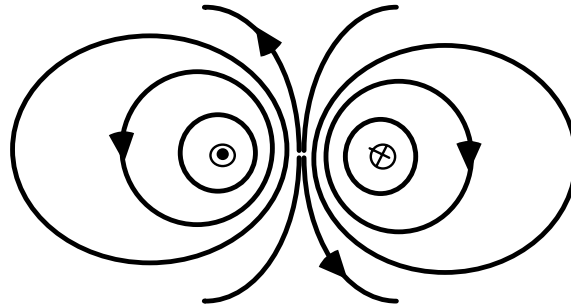
3.4. MAGNETIC DIPOLE

3.4.A. MAGNETIC DIPOLES

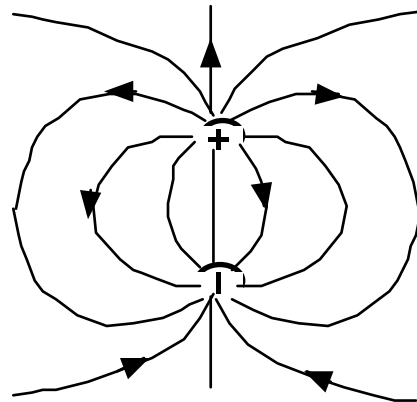
SIMPLE DIPOLE: a loop of current I , area a

magnetic field of simple loop

dipole moment is $\mathbf{m} \equiv I \mathbf{a}$



Far field has same pattern
as electric dipole



more generally, $\mathbf{m} = \frac{1}{2} \int d^3\mathbf{R} \mathbf{R} \times \mathbf{J}(\mathbf{R})$, which is independent of origin.

Dipole magnetic vector potential and field

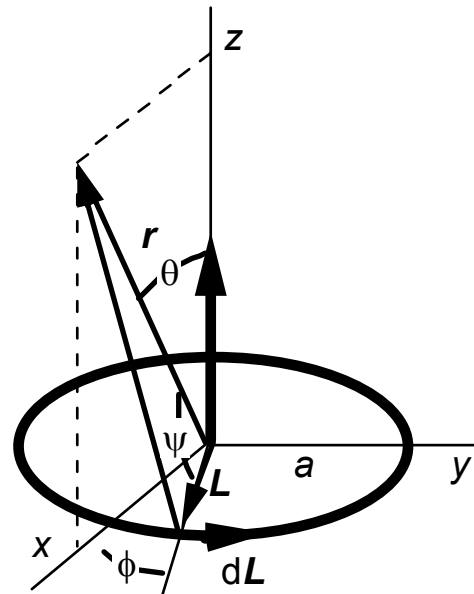
$$\mathbf{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{r}}{r^2}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} [3(\mathbf{m} \cdot \hat{r})\hat{r} - \mathbf{m}] = \frac{\mu_0 m}{4\pi r^3} (2\hat{r} \cos \theta + \hat{z} \sin \theta) \text{ for } r \rightarrow \infty, \mathbf{m} \parallel \hat{z}$$

SIMPLE LOOP: a loop
of current I , area $dS = \pi a^2$
compute $\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int d\mathbf{L} \times (\mathbf{r} - \mathbf{R}) / |\mathbf{r} - \mathbf{R}|^3$

coordinates: choose $y = 0$,
 \mathbf{L} in xy plane $\Rightarrow A_z = 0$,
 $d\mathbf{L} = a d\phi \hat{\phi} = a d\phi (-\hat{x} \sin\phi + \hat{y} \cos\phi)$
symmetry $\Rightarrow B_y = 0$

dipole moment is
 $\mathbf{m} = I d\mathbf{S} = I \pi a^2 \hat{z}$



numerator: $d\mathbf{L} \times (\mathbf{r} - \mathbf{L}) = d\mathbf{L} \times \mathbf{r} - d\mathbf{L} \times \mathbf{L}$; substitute $\mathbf{r} = x \hat{x} + z \hat{z}$
 $\rightarrow d\mathbf{L} \times (\mathbf{r} - \mathbf{L}) = a d\phi \{x(-\hat{z} \cos\phi) + z(-\hat{y} \sin\phi + \hat{x} \cos\phi)\} + a^2 d\phi \hat{z}$
the \hat{y} component integrates to zero by symmetry, so we can use

$$d\mathbf{L} \times (\mathbf{r} - \mathbf{L}) \rightarrow a d\phi \{ (z \cos\phi) \hat{x} + (a - x \cos\phi) \hat{z} \}$$

denominator: $|\mathbf{r} - \mathbf{L}|^2 = r^2 + a^2 - 2a r \cos\psi = r^2 + a^2 - 2a x \cos\phi$
since $r \cos\psi =$ projection of \mathbf{r} along $\mathbf{L} = x \cos\phi$

far away: look at $r \gg a$, let $\epsilon = a/r \ll 1$, then

$$\frac{1}{|\mathbf{r} - \mathbf{L}|^3} = \frac{1}{r^3} (1 - 2\epsilon \frac{x}{r} \cos\phi + \epsilon^2)^{-3/2} \approx \frac{1}{r^3} (1 + 3\epsilon \frac{x}{r} \cos\phi)$$

$$\Rightarrow \mathbf{B}(\mathbf{r}) \approx \frac{\mu_0 I a}{4\pi r^3} \int_0^{2\pi} d\phi \{ (z \cos\phi) \hat{x} + (a - x \cos\phi) \hat{z} \} (1 + 3\epsilon \frac{x}{r} \cos\phi)$$

now $\int_0^{2\pi} d\phi = 2\pi$, $\int_0^{2\pi} d\phi \cos\phi = 0$, $\int_0^{2\pi} d\phi \cos^2\phi = \pi$, $\frac{x}{r} = \sin\theta$, $\frac{z}{r} = \cos\theta$

$$\mathbf{B}(\mathbf{r}) \approx \frac{\mu_0 I a}{4r^3} \{ 3\epsilon \frac{xz}{r} \hat{x} + (2a - 3\epsilon \frac{x^2}{r}) \hat{z} \} = \frac{\mu_0 I a^2}{4r^3} \{ 3\sin\theta \cos\theta \hat{x} + (2 - 3\sin^2\theta) \hat{z} \}$$

but $\hat{r} = \sin\theta \hat{x} + \cos\theta \hat{z}$, so $\mathbf{B}(\mathbf{r}) \approx \frac{\mu_0 I a^2}{4r^3} \{ 3\cos\theta \hat{r} - \hat{z} \}$, or

$$\mathbf{B}(\mathbf{r}) \approx \frac{\mu_0}{4\pi r^3} \{ 3(\mathbf{m} \cdot \hat{r}) \hat{r} - \mathbf{m} \}$$

3.4.B. TORQUE ON A DIPOLE

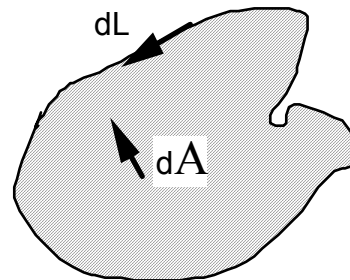
dipole moment $\mathbf{m} = I \mathbf{A}$ Force on $dL(\mathbf{R})$ is

$$d\mathbf{F}(\mathbf{R}) = I d\mathbf{L}(\mathbf{R}) \times \mathbf{B}$$

Torque on $dL = \mathbf{R} \times d\mathbf{F}(\mathbf{R})$

Total torque

$$\boldsymbol{\tau} = I \oint \mathbf{R} \times (d\mathbf{L}(\mathbf{R}) \times \mathbf{B})$$

vector identity 2: $\boldsymbol{\tau} = I \oint [d\mathbf{L}(\mathbf{r})(\mathbf{r} \cdot \mathbf{B}) - \mathbf{B} (d\mathbf{L}(\mathbf{r}) \cdot \mathbf{r})]$ last term integrates out: $\mathbf{r} = \frac{1}{2} \nabla(r^2)$ consider part of $\boldsymbol{\tau}$ proportional to B_x :

$$\begin{aligned} d\mathbf{L}(\mathbf{r}) = d\mathbf{r} \Rightarrow \quad & I B_x \oint d\mathbf{L}(\mathbf{r}) \times \mathbf{x} = I \oint d\mathbf{r} \times \mathbf{x} \\ & = I B_x \oint (\hat{x} dx + \hat{y} dy + \hat{z} dz) \times \mathbf{x} \\ & = I B_x (\hat{x} \oint x dx + \hat{y} \oint x dy + \hat{z} \oint x dz) \\ & = I B_x (\hat{x} (0) + \hat{y} A_z - \hat{z} A_y) \end{aligned}$$

compare $\mathbf{m} \times \mathbf{B} = I \mathbf{A} \times \mathbf{B}$, part proportional to B_x is

$$I (\hat{y} A_z - \hat{z} A_y) B_x \quad \checkmark$$

3.5. MAGNETIZATION

INTRINSIC MAGNETIC MOMENTS

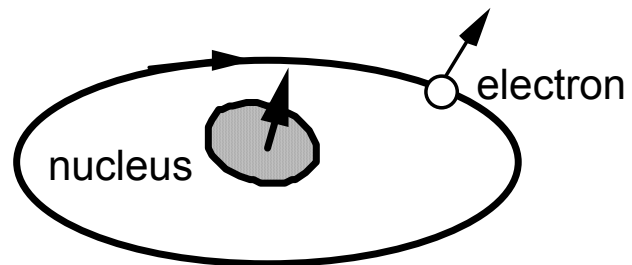
Properties of elementary particles include:

- Electric charge (quantized in units $\frac{1}{3} e$)
- Mass = energy in rest frame/ c^2 (no known natural unit)
- Angular momentum (quantized in units $\hbar / 2 = h / 4\pi$)
- Magnetic moment (no known natural unit)
 - always parallel to the intrinsic angular momentum
 - both are relativistic effects, arise in Dirac's theory

NUCLEI AND ATOMS HAVE MAGNETIC MOMENTS

They arise partly from the intrinsic moments of the electrons and quarks,

and partly from their motion in little current loops.



Result: every material has a magnetic moment per molecule m

Magnetization $\mathbf{M} = \mathbf{m} N$,

where \mathbf{m} = magnetic moment per molecule

N = number of molecules per unit volume

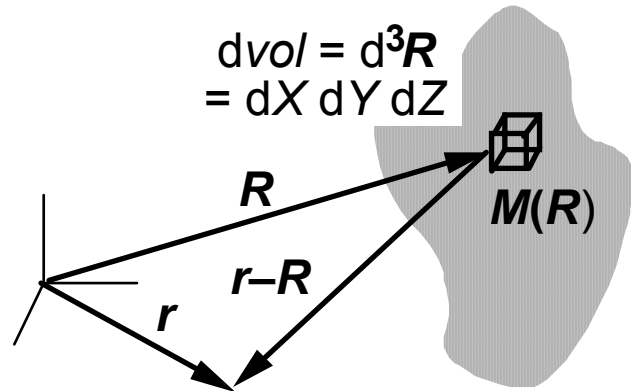
dipole \mathbf{m} at \mathbf{R} →

magnetic field

$$\mathbf{B}_m(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3(\vec{m} \cdot \hat{r}_R)\hat{r}_R - \vec{m}}{|\vec{r} - \vec{R}|^3}$$

Where $\hat{r}_R = \frac{\vec{r} - \vec{R}}{|\vec{r} - \vec{R}|}$

magnetization $\mathbf{M}(\mathbf{R})$ →



$$\text{field } \mathbf{B}_M(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3R \frac{\mu_0}{4\pi} \frac{3(\vec{M} \cdot \hat{r}_R)\hat{r}_R - \vec{M}}{|\vec{r} - \vec{R}|^3}$$

Note \mathbf{M} takes account of both

- the intrinsic moments
- motion of the bound charges.

EFFECTIVE CURRENTS

calculate (with some difficulty)

$$\nabla \times \mathbf{B}_M(\mathbf{r}) = \mu_0 \nabla \times \mathbf{M}(\mathbf{r}) + \mu_0 \mathbf{J}_{\text{pol}}(\mathbf{r})$$

The field can be thought of as arising from an "effective current" $\mathbf{J}_{\text{pol}}(\mathbf{r})$

In addition to the fields and potentials due to \mathbf{M} , there are other sources:

- free currents \mathbf{J}_{free}
- polarization current $\mathbf{J}_{\text{pol}} = \partial \mathbf{P} / \partial t$ due to the electric dipole density \mathbf{P}

see "Surface Polarization"(in Notes on Electrodynamics)

Conclusion: the magnetic field arises

from an effective current density $\mathbf{J}_{\text{effective}} = \nabla \times \mathbf{M}$

and the surface current density $\alpha_{\text{effective}} = \mathbf{M} \times \hat{n}$

in addition to the free current density \mathbf{J}_{free}

and the polarization current $\mathbf{J}_{\text{pol}} = \partial \mathbf{P} / \partial t$

3.5.A. MAGNETIC FIELD STRENGTH H

Inside magnetic materials $\nabla \times \mathbf{B} / \mu_0 = \mathbf{J}_{\text{total}} = \mathbf{J}_{\text{free}} + \nabla \times \mathbf{M}$

so

$$\mathbf{J}_{\text{free}} = \nabla \times \mathbf{H} \quad \text{where} \quad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \quad \left(\text{units } \frac{\text{amperes}}{\text{meter}} \right)$$

compare dielectrics, $\rho_{\text{free}} = \nabla \cdot \mathbf{D}$, $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$

integral form: use Stokes' theorem \Rightarrow

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int \mathbf{J} \cdot d\mathbf{A} = I_{\text{free}} \quad \text{the free current enclosed}$$

at the surface of a magnetic material

• integrate $\nabla \cdot \mathbf{B} = 0$

over thin Gaussian volume

$$\int dV \nabla \cdot \mathbf{B} = 0$$

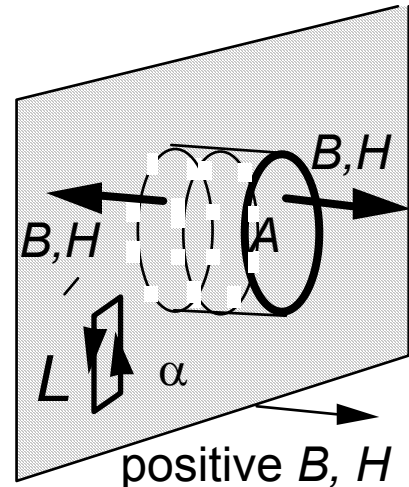
(Stokes) $\int dS \mathbf{B} \cdot \hat{\mathbf{n}}_{\text{normal}} = 0$

$$S(\mathbf{B}_{\text{outer} \perp} - \mathbf{B}_{\text{inner} \perp}) = 0$$

• loop crossing surface

$$\oint \mathbf{H} \cdot d\mathbf{L} = I_{\text{free}}$$

$$L(\mathbf{H}_{\text{outer} \parallel} - \mathbf{H}_{\text{inner} \parallel}) = I_{\text{free}}$$



\Rightarrow \mathbf{B}_{\perp} , \mathbf{H}_{\parallel} are continuous

see "Core Example"

DEFINITIONS AND NOTATION (linear response)

• Magnetic susceptibility χ_m

$$\mathbf{M} = \chi_m \mathbf{H}$$

• Permeability μ

$$\mathbf{B} = \mu \mathbf{H}$$

• Relative permeability $\mu_{\text{rel}} = \mu / \mu_0 = 1 + \chi_m$

• Main equation

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

warning: ferromagnetic materials are not linear

CORE EXAMPLE

Wind a long solenoid on a magnetic core, $N = N' L$ turns

Ampere: $\oint \mathbf{H} \cdot d\mathbf{L} = I_{\text{free}}$

symmetries \Rightarrow choose path shown

$$(H_{\text{in}} - H_{\text{out}})_{\parallel} L = N'I$$

independent of how far outside

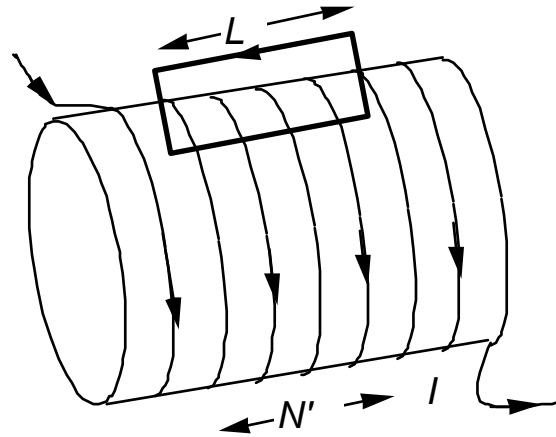
$$\Rightarrow H_{\text{out}} \approx 0 \Rightarrow H_{\text{in}} \approx N'I$$

independent of where inside!

$$B_{\text{in}} = \mu H_{\text{in}} \approx \mu N'I > \mu_0 H_{\text{in}}$$

contrast dielectric capacitor,

$$E_{\text{in}} = D_{\text{in}}/\epsilon < D_{\text{in}}/\epsilon_0$$



3.5.B. FERROMAGNETISM

A ferromagnet consists of microscopic domains, in each of which the atomic magnetic moments are lined up.

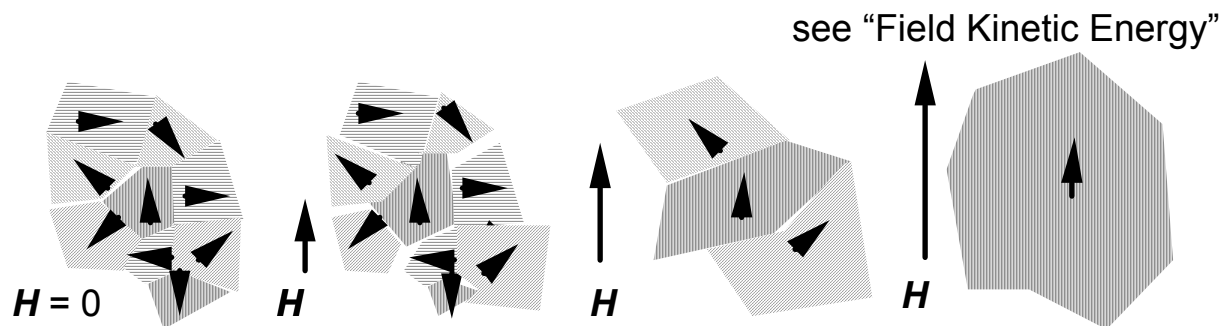
- a. If a ferromagnet is heated to a high enough temperature $T \geq T_{\text{Curie}}$ and then cooled, the domains are randomly oriented.

Thus the net magnetization is zero.

- b. An applied external field makes

- the domains aligned with the field grow,
- the domains with the wrong orientation shrink.

This causes the magnetic (kinetic) energy to increase.



- c. When most of the domains are roughly aligned, a still stronger field causes the domains to align completely.

- d. Application of a larger external field produces no additional magnetization.

see "Hysteresis"

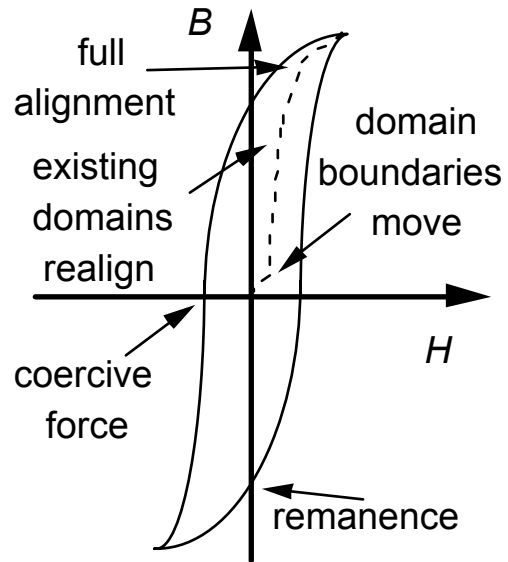
HYSTERESIS

Once a ferromagnet has been placed in a large field, it remains magnetized even when the external field is removed, because the alignment of the domains has increased the magnetic kinetic energy.

The residual field is the *remanence*.

To decrease the internal field to zero, an external field must be applied.

The needed external field is the *coercive force*.



Energy stored in magnet = $\int d\mathbf{B} \cdot \mathbf{H}$ = area under $B - H$ curve.

In each cycle, energy is lost: $E_{\text{lost}} = \text{area inside curve}$.

THE BAR MAGNET

with uniform magnetization $\mathbf{M} \parallel$ to its sides

is equivalent to a solenoid with

a current density $N'I = M$ equal to the surface current.

Its *external* field flows
from north pole to south.

\mathbf{B} forms closed loops
through the interior;

outside the magnet, $\mathbf{H} = \mathbf{B}$,
but inside the magnet
 \mathbf{H} is opposed to \mathbf{B} !!!

- $\oint \mathbf{H} \cdot d\mathbf{L} = 0$, consider integral
along line of \mathbf{B} .

- \mathbf{H}_{\parallel} must be continuous through the side of the magnet,
where \mathbf{B} reverses direction.

