Sources
$$\varepsilon_0 \nabla \cdot \mathbf{E} = \rho_{total} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}_{total}$$
$$\varepsilon_0 \int \mathbf{E} \cdot d\mathbf{S} = Q_{enclosed} \qquad \oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{enclosed}$$

Application to symmetric situations: make integrand piecewise constant

Static Constraints
$$\nabla \times \mathbf{E} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$
$$\oint \mathbf{E} \cdot d\mathbf{L} = 0 \qquad \int \boldsymbol{B} \cdot d\boldsymbol{S} = 0$$

Surface
$$\varepsilon_0 \Delta \mathbf{E} = \sigma_{total}\, \hat{n} \qquad \Delta \mathbf{B} = \mu_0 \mathbf{K}_{total} \times \hat{n}$$

Charge conservation
$$\nabla \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$$

General Solutions by superposition
$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \int d^3\boldsymbol{R}\, \frac{(\boldsymbol{r}-\boldsymbol{R})\rho_{total}(\boldsymbol{R})}{|\boldsymbol{r}-\boldsymbol{R}|^3} \qquad \boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int d^3\boldsymbol{R}\, \frac{\boldsymbol{J}_{total}(\boldsymbol{R}) \times (\boldsymbol{r}-\boldsymbol{R})}{|\boldsymbol{r}-\boldsymbol{R}|^3}$$

Far Fields
$$\boldsymbol{E}(\boldsymbol{r}) \approx \frac{q_{total}\hat{\boldsymbol{r}}}{4\pi\varepsilon_0 r^2} + \frac{3(\boldsymbol{p}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{p}}{4\pi\varepsilon_0 r^3} \qquad \boldsymbol{B}(\boldsymbol{r}) \approx \mu_0 \frac{3(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{m}}{4\pi r^3}$$

Moments
$$\boldsymbol{p} = \int d^3R\ \boldsymbol{R}\, \rho(\boldsymbol{R}) \qquad \boldsymbol{m} = \frac{1}{2} \int d^3R\ \boldsymbol{R} \times \boldsymbol{J}(\boldsymbol{R})$$

Paradigms of Moments
$$\boldsymbol{p} = q\, \boldsymbol{d} \qquad \boldsymbol{m} = I\, \boldsymbol{A}$$

Bound Densities
$$\nabla \cdot \boldsymbol{P} = -\rho_{bound} \qquad \nabla \times \boldsymbol{M} = \boldsymbol{J}_{bound}$$
$$\sigma_{bound} = \boldsymbol{P} \cdot \hat{n} \qquad \boldsymbol{K}_{bound} = \boldsymbol{M} \times \hat{n}$$

Auxiliary Fields
$$\boldsymbol{D} \equiv \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} \qquad \boldsymbol{H} \equiv \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M}$$

Source of Aux. Field
$$\nabla \cdot \boldsymbol{D} = \rho_{free} = \rho_{total} - \rho_{bound} \qquad \nabla \times \boldsymbol{H} = \boldsymbol{J}_{free} = \boldsymbol{J}_{total} - \boldsymbol{J}_{bound}$$

Boundary Conditions
$$(\Delta \boldsymbol{D})_\perp = \sigma_{free}\hat{\boldsymbol{n}} \qquad (\Delta \boldsymbol{H})_{||} = \boldsymbol{K}_{free} \times \hat{\boldsymbol{n}}$$
$$(\Delta \boldsymbol{E})_{||} = 0 \qquad (\Delta \boldsymbol{D})_\perp = 0$$

Linear Response in medium
$$\boldsymbol{P} = \chi_{el}\, \boldsymbol{E}\, ,\ \boldsymbol{D} = \varepsilon \boldsymbol{E} \qquad \boldsymbol{M} = \chi_{mag}\, \boldsymbol{H}\, ,\ \boldsymbol{B} = \mu \boldsymbol{H}$$

Numerical values $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$ $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

Potential energy density = $\frac{1}{2}\ \boldsymbol{D}\cdot\boldsymbol{E}$ Lorentz force $\boldsymbol{F}_q = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

Capacitance $Q = CV$ Force on Circuit $\boldsymbol{F} = I \oint d\boldsymbol{l} \times \boldsymbol{B}$

Potential energy $U = 1/2QV$

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