Dynamics Review

differential	MAXWELL'S EQUATIONS	integral
$\nabla\cdot\boldsymbol{B} = 0$		$\oint d\mathbf{A}\cdot\boldsymbol{B} = 0$
$\nabla\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t} = 0$		$\oint d\boldsymbol{L}\cdot\boldsymbol{E} = -\frac{d}{dt}\int d\mathbf{A}\cdot\boldsymbol{B} \equiv -\frac{d\Phi}{dt}$
$\nabla\cdot\boldsymbol{E} = \rho/\varepsilon_o$		$\oint d\mathbf{A}\cdot\boldsymbol{E} = Q_{enclosed}/\varepsilon_o$
$\nabla\times\boldsymbol{B} - \frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t} = \mu_o\boldsymbol{J}$		$\oint d\boldsymbol{L}\cdot\boldsymbol{B} = \mu_o\int d\mathbf{A}\cdot(\boldsymbol{J} + \varepsilon_o\frac{\partial\boldsymbol{E}}{\partial t})$
$\nabla\cdot\boldsymbol{J} + \partial\rho/\partial t = 0$		$\oint d\mathbf{A}\cdot\boldsymbol{J} = -\partial Q_{enclosed}/\partial t$

- Polarization density of electric dipole moments $\boldsymbol{P}$
- Polarization density of magnetic dipole moments $\boldsymbol{M}$
- Displacement field $\boldsymbol{D} \equiv \varepsilon_o\boldsymbol{E} + \boldsymbol{P}$
- Polarization charge $\rho_{bound} \equiv -\nabla\cdot\boldsymbol{P}$
- Free charge $\rho_{free} \equiv \rho - \rho_{bound}$
- Polarization current $\boldsymbol{J}_{pol} \equiv \partial\boldsymbol{P}/\partial t$
- Current $\boldsymbol{J}_{free} \equiv \boldsymbol{J} - \boldsymbol{J}_{pol} - \nabla\times\boldsymbol{M}$
- Field strength $\boldsymbol{H} \equiv \boldsymbol{B}/\mu_o - \boldsymbol{M}$

differential	MAXWELL IN MATTER	integral
$\nabla\cdot\boldsymbol{B} = 0$		$\oint d\mathbf{A}\cdot\boldsymbol{B} = 0$
$\nabla\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t} = 0$	(Faraday)	$\oint d\boldsymbol{L}\cdot\boldsymbol{E} = -\frac{d}{dt}\int d\mathbf{A}\cdot\boldsymbol{B}$
$\nabla\cdot\boldsymbol{D} = \rho_{free}$	(Gauss)	$\oint d\mathbf{A}\cdot\boldsymbol{D} = Q_{free}$
$\nabla\times\boldsymbol{H} - \frac{\partial\boldsymbol{D}}{\partial t} = \boldsymbol{J}_{free}$	(Ampère)	$\oint d\boldsymbol{L}\cdot\boldsymbol{H} = \int d\mathbf{A}\cdot(\boldsymbol{J}_{free} + \frac{\partial\boldsymbol{D}}{\partial t})$
$\nabla\cdot\boldsymbol{J}_{free} + \partial\rho_{free}/\partial t = 0$	(charge cons)	$\oint d\mathbf{A}\cdot\boldsymbol{J}_{free} = -\partial Q_{free}/\partial t$

Linear media: $\boldsymbol{P} = \chi_e\boldsymbol{E}$, $\boldsymbol{D} = \varepsilon\boldsymbol{E} = (1+\chi_e)\varepsilon_o\boldsymbol{E}$

(but ferromagnets aren't linear) $\mathbf{M} = \chi_m\mathbf{H}$, $\mathbf{B} = \mu\mathbf{H} = (1+\chi_m)\mu_o\mathbf{H}$

Field Energy

$d\rho_{MagE}(\boldsymbol{r}) = \boldsymbol{H}(\boldsymbol{r})\cdot d\boldsymbol{B}(\boldsymbol{r})$, $d\rho_{ElecE}(\boldsymbol{r}) = \boldsymbol{E}(\boldsymbol{r})\cdot d\boldsymbol{D}(\boldsymbol{r})$, $\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{H}$, $\nabla\cdot\boldsymbol{S} + \partial\rho_{Energy}/\partial t = 0$

Inductance Mutual inductance $\Phi_{ab} = M_{ab} I_a$, Self inductance $\Phi = L\, I$

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(revised by Yun-Shik Lee)

Electromagnetic Waves

Plane waves in uniform space:

$$\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \text{ and } \boldsymbol{B}(\boldsymbol{r},t)=\boldsymbol{B}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}=\frac{1}{c}\hat{\boldsymbol{k}}\times\boldsymbol{E}(\boldsymbol{r},t)$$

dispersion relation $k^2=\mu\varepsilon\omega^2+i\mu\sigma\omega,\ k=k_R+ik_I$

wavelength $\lambda = 2\pi\,\bar{\lambda} \equiv \dfrac{2\pi}{\mathrm{Re}k}$ $\qquad \bar{\lambda} = 1/k_R$

definitions of wave quantities	insulator (σ=0)	free space
phase velocity $v_p \equiv \omega/k_R$	$v_p = \dfrac{1}{\sqrt{\varepsilon\mu}}$	$v_p = c$
index of refraction $n \equiv \dfrac{c}{v_p}$	$n = \sqrt{(\varepsilon_r \mu_r)}$	$n = 1$
Attenuation length $\delta \equiv 1/\,\mathrm{Im}\,k$ "skin depth"		$\delta = \infty$ (no attentuation)

EM Waves at Plane Boundary match on interface: $(\boldsymbol{k}_I)\times\hat{n} = (\boldsymbol{k}_R)\times\hat{n} = (\boldsymbol{k}_T)\times\hat{n} = \boldsymbol{k}\times\hat{n}$

$k_1\sin\theta_I = k_1\sin\theta_R$

$\Rightarrow \theta_I = \theta_R = \theta_1$

law of reflection

wave vectors coplanar

$k_1\sin\theta_1 = k_2\sin\theta_R$

$\Rightarrow n_1\sin\theta_1 = n_2\sin\theta_2$

Snell's law

$$E_{Tm\perp} = E_{Im\perp} \times \frac{2\,n_1\cos\theta_1}{n_1\cos\theta_1+n_2\cos\theta_2} \qquad E_{Tm\parallel} = E_{Im\parallel} \times \frac{2\cos\theta_1/n_2}{\cos\theta_1/n_1+\cos\theta_2/n_2}$$

$$E_{Rm\perp} = E_{Im\perp} \times \frac{n_1\cos\theta_1-n_2\cos\theta_2}{n_1\cos\theta_1+n_2\cos\theta_2} \qquad E_{Rm\parallel} = E_{Im\parallel} \times \frac{\cos\theta_2/n_2-\cos\theta_1/n_1}{\cos\theta_1/n_1+\cos\theta_2/n_2}$$

Brewster angle: $\theta_1+\theta_2 = \pi/2$

Potential Fomulation

fields from potential: $\boldsymbol{B} = \nabla\times\boldsymbol{A}\ ,\ \boldsymbol{E} = -\nabla V - \dfrac{\partial\boldsymbol{A}}{\partial t}$

retarded potential $\mathbf{A}(\mathbf{r}) = \dfrac{\mu_o}{4\pi}\displaystyle\int d^3\boldsymbol{R}\ \dfrac{\mathbf{J}(\boldsymbol{R},T=t-|\boldsymbol{R}-\boldsymbol{r}|/c)}{|\boldsymbol{R}-\boldsymbol{r}|}$ $\quad \mathbf{J} = (\boldsymbol{J}, c\rho)\ ,\ \mathbf{A}(\boldsymbol{r},t) = (\boldsymbol{A}(\boldsymbol{r},t), V(\boldsymbol{r},t)/c)$

Electric Dipole Radiation

electric dipole $\boldsymbol{p}(t)=\boldsymbol{p}_0 e^{i\omega t}$, $\underline{s \ll \lambda,\ r}$ $\Rightarrow V(\mathbf{r}) \approx \dfrac{\boldsymbol{r}\cdot\boldsymbol{p}(t-r/c)}{4\pi\varepsilon_o r^2}\left(\dfrac{1}{r}+ik\right)$ = static + radiation

$\boldsymbol{E}_{\mathrm{rad}}(\boldsymbol{r},t) = e^{i\omega(t-r/c)} \times \dfrac{1}{4\pi\varepsilon_o}\dfrac{k^2}{r}\left(\boldsymbol{p}_o - \dfrac{(\boldsymbol{r}\cdot\boldsymbol{p}_o)\boldsymbol{r}}{r^2}\right)$, $\underline{\lambda \ll r}$

In spherical coordinate ($\boldsymbol{p}(t)\ //\ \hat{z}$) $E \propto \dfrac{\sin\theta}{r}$, $\boldsymbol{E}(\boldsymbol{r},t)\ //\ -\hat{\theta}$, $\boldsymbol{B}(\boldsymbol{r},t)\ //\ -\hat{\phi}$, and $\boldsymbol{S}(\boldsymbol{r},t)\ //\ \hat{r}$