

PH 451: Capstone in Quantum Mechanics

Homework 7

Due 2/22/08

1. For two non-interacting distinguishable particles in a 1-d infinite square well potential (width a), find the allowed energy levels that are less than $30 \frac{\pi^2 \hbar^2}{2ma^2}$. Consider the cases where (a) $m_1 = m_2 = m$ and (b) $m_1 = m$ and $m_2 = 2m$. Find an appropriate numbering scheme for the levels, and make an ordered table, noting any degeneracy. Is the degeneracy accidental?
2. Use Maple or Mathematica to plot the probability density for two non-interacting particles in a 1-d harmonic oscillator potential. One of the particles is in the single particle ground state and the other is in the single particle first excited state. Do this for (a) distinguishable particles (of the same mass) (b) identical fermions with an antisymmetric wavefunction and (c) identical bosons with a symmetric wave function. In each case write the system wave function and discuss the important features of your plots.
3. Consider two non-interacting particles, each of mass m , in an infinite square well. If one particle is in the state $|\psi_n\rangle$ and the other in the state $|\psi_k\rangle$ ($n \neq k$), calculate $\langle (x_1 - x_2)^2 \rangle$, assuming (a) they are distinguishable particles, (b) they are identical bosons, and (c) they are identical fermions. Use the bra-ket notation as far as you can, but you will have to do some integrals.